Ma 3113 **Introduction to Linear Algebra** Effective Fall, 2002

Text: *Linear Algebra and Its Applications*, 3rd Edition, by David C. Lay

This course will present the main concepts and terminology of linear algebra that play an essential role in science and engineering. The syllabus below, essentially Lay's syllabus B, below outlines a core of topics based on the recommendations of the NSF-funded Linear Algebra Curriculum Study Group. Specifically, LACSG suggests that Introduction to Linear Algebra should be viewed as a service course for the various disciplines that employ techniques of linear algebra. Applications illustrating the pervasive use of linear algebra in engineering and the sciences should be included in the course. Utilization of technology is strongly encouraged, and the schedule below accordingly allows a significant amount of time for supplementary topics of the instructor's choice beyond the essential topics of the core.

There is a supporting web page available to faculty and students at http://www.laylinalgebra.com which contains supplementary materials and computer applications, including Mathematica files and data files for exercises marked [M]. Additional manuals and resources will also be available in the main office.

Another on-line source of projects is the ATLAST site:
http://www.umassd.edu/SpecialPrograms/Atlast/welcome.html

In the following, 1 hour equals 50 minutes.

Chapter

1. Linear Equations Sections 1.1–1.5, 1.7–1.9 . . . . 7 hours
Important Skills and Ideas Solve a system by row reduction. Determine when a system is consistent. Write the general solution in parametric vector form. Describe existence or uniqueness of solutions in terms of pivot positions. Determine when a homogeneous system has nontrivial solutions. Describe the solution set of a nonhomogeneous system as a translation of the solution set of the corresponding homogeneous system. Determine when a vector is in the subset spanned by specified vector. Exhibit a vector as a linear combination of vectors. Determine whether the columns of an $m \times n$ matrix span $\mathbb{R}^m$. Determine if the columns are linearly independent. Determine whether a set of vectors in $\mathbb{R}^n$ are linearly independent. Find the standard matrix for a linear transformation. Describe the action of operators on $\mathbb{R}^2$, in particular dilations, rotations and projections. Determine if a linear transformation is one-to-one or onto.

Applications/projects In addition to Lay's "tool box", some of which seems outdated, there is primer on *Mathematica* in general and basic linear algebra commands located on the local Novell directory Jasper\Labs\Homes\Math. For example, there are commands `LinearSolve` and `NullSpace` that return respectively a solution of the equation $Ax = b$ and a basis for the solution set of the the corresponding homogeneous system. A mini project using these commands is a good way to reinforce the material on structure of solution sets in Section 1.5. Lay has two excellent projects that can be used following Section 1.2: 1) Interpolating polynomials and (2) Splines. Lay provides a third application following Section 1.9.

2. Matrix Algebra Sections 2.1–2.5, 2.8, 2.9 . . . . 7 hours
Important Skills and Ideas Know the definition and properties of matrix product. Know the relation between matrix multiplication and composition of transformations. Compute the

inverse of a matrix using row reduction. Use a matrix inverse to solve a system of equations. Use matrix algebra to solve matrix equations. Use the Invertible Matrix Theorem to connect various properties of square matrices. Compute entries in a product of partitioned matrices. Construct an LU factorization of a matrix A, and use such a factorization to solve a system $Ax = b$. Know the subspaces of $\mathbb{R}^2$ and $\mathbb{R}^3$; interpret span$\{u\}$ and span$\{u, v\}$ geometrically. Determine if a set of vectors spans $\mathbb{R}^n$. Determine if a set of vectors is a basis for a subspace W of $\mathbb{R}^n$. Find the coordinate vector of a vector relative to a given basis. Find bases for nul(A) and col(A). Find the dimension of a subspace. Know the Rank Theorem.

Applications/projects Lay provides several projects involving matrix products and factorizations. One can also introduce Markov processes and investigate steady state phenomena numerically. This topic can be revisited after the introduction of eigenvalues and dynamical systems.

3. Determinants Sections 3.1–3.2 . . . . 2 hours
Important Skills and Ideas Compute det(A) using a cofactor expansion, taking advantage of 0's. Compute a determinant by triangularization. Know the properties of determinants with respect to inverses, products, transposes and scalar multiples.

5. Eigenvalues and Eigenvectors Sections 5.1–5.3, 5.5–5.7 . . . . 6 hours
Important Skills and Ideas Use the characteristic polynomial to find eigenvalues of a matrix. Find a basis for an eigenspace. Determine if a matrix is diagonalizable. Diagonalize a matrix, and show how to use the diagonalization to compute powers of the matrix. For A diagonalizable, solve the recursive equation $x_{k+1} = Ax_k$ in terms of the eigenvalues and eigenvectors of A. Know the significance of a dominant eigenvalue for long-term behavior of a dynamical system. Know that complex eigenvalues and eigenvectors of a real matrix occur in conjugate pairs. Express $x \mapsto \begin{pmatrix} a & -b \\ b & a \end{pmatrix} x$ as a rotation and a dilation. If A is a real 2×2 matrix with complex eigenvalues, express A in the form $A = P \begin{pmatrix} a & -b \\ b & a \end{pmatrix} P^{-1}$, and describe the behavior of the dynamical system $x_{k+1} = Ax_k$. Solve a first order linear system $x'(t) = Ax(t)$ for A diagonalizable, and, in the 2×2 case, describe trajectories of solutions.

Applications/projects Besides the [M] exercises, there are routine exercises on diagonalization located in the Jasper\Labs\Homes\Math directory of our Novell network. There are also projects there regarding discrete dynamical systems, applications to population dynamics, stochastic matrices and first order systems of linear differential equations

6. Orthogonality and Least-Squares Sections 6.1–6.6 . . . . 5 hours
Important Skills and Ideas Compute the length of a vector, distance between vectors. Normalize a vector. Check a set for orthogonality. Express a vector v as $v = v_1 + v_2$ where v_1 is parallel to a given vector u and v_2 is orthogonal to u. Know the identity col$(A)^\perp = $ nul(A^T), and compute the orthogonal complement of a subspace. Use the Gram-Schmidt process to obtain an orthogonal basis for a subspace. Compute the QR factorization of a matrix and use it to solve linear systems. Compute the orthogonal projection of a vector onto a subspace. Find the distance of a vector to a subspace. Find a least squares solution of a system $Ax = b$, and compute the least squares error. Find the least squares line (or other curve) that best fits given data.

Applications/projects Besides the [M] exercises, Lay gives a very nice case study regarding least squares solutions, Section 6.5, as well as a project on QR decomposition, 6.4. There are other problems and data sets regarding linear models at Jasper\Labs\Homes\Math.

4. Function Spaces Sections 4.1 and parts of 4.3, 4.5, 6.7 and 6.8 . . . . 5 hours
Important Skills Know the definition of a vector space and the notions of linear independence and dimension in this setting. Know the dimension of P_n, the space of polynomials of degree $\leq n$. Find the coordinate vector of a polynomial relative to a given basis. Determine if a set of polynomials is linearly independent. Compute $\|f\|$ and $\langle f, g\rangle$ in $C[a, b]$ with respect to the usual integral inner product. Compute the best approximation of a function by polynomials of degree $\leq n$, and know how this relates to orthogonal projections. Find the n^{th} order Fourier approximation of a function on $[0, 2\pi]$.

Applications/projects Some exercises involving orthogonal polynomials and Fourier approximations are located at Jasper\Labs\Homes\Math.

Total: 32 hours

Of the remaining time, about 13 hours, 6–7 hours should be allotted for applications and projects. In addition to projects mentioned above, instructors may also cover supplementary sections from the text including **2.8** Computer Graphics, **4.8** Difference Equations, and **7.1** Diagonalization of Symmetric Matrices.