

DEPARTMENT OF MATHEMATICS AND STATISTICS
MISSISSIPPI STATE UNIVERSITY

SPENCER B. MURRAY AND ARTHUR OLLIVIER LECTURE

Cellular Automata

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and

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McCool 100

Abstract. Cellular automata on graphs were introduced by von Neumann over sixty years ago. Somewhat vaguely, these are deterministic processes on graphs defined by local update rules. One of their simplest examples is bootstrap percolation on a graph G with threshold r . In this process we start with an ‘initial configuration’, a partition of the vertex set into ‘healthy’ and ‘infected’ ‘sites’, and let the infection ‘spread’ according to the following simple rule: an infected site remains infected for ever, and at each time step a healthy vertex becomes infected if at least r of its neighbours are infected. If eventually all vertices become infected then we say that the process ‘percolates’. Whether a process percolates or not depends only on the initial configuration. In a basic question concerning bootstrap percolation, we start with a random set of infected sites in which each site is infected with probability p , independently of the others, and should like to determine the ‘critical probability’ $p_c(G, r)$, the value of p for which the probability of percolation is $1/2$.

In 1988, Aizenman and Lebowitz gave bounds on the critical probability $p_c([n]^d, 2)$ for percolation on the d -dimensional grid $[n]^d$ with n^d sites, as $n \rightarrow \infty$. These results were sharpened and generalized by a number of people, including Cerf, Cirillo, Manzo and Holroyd. I shall review some of these results and shall present a number of recent advances I have made jointly with Balogh and Morris.

In the case of probabilistic automata the update rules are similar to those in deterministic automata, but are obeyed only with some probability. Although these processes have been studied for close to forty years, there are very few rigorously proved theorems about them. In this talk I shall review some of these results, and shall present a number of recent theorems obtained jointly with Balister, Johnson and Walters. Our results go far beyond anything obtainable by simulation, and provide evidence for the existence of critical probabilities for the random majority-vote cellular automata on tori.

A reception will follow Dr. Bollobás’s talk.

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